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MODEL BASED DESIGN OF PIEZOELECTRIC GENERATORS UTILISING GEOMETRICAL AND MATERIAL PROPERTIES

Björn Richter*
Mechatronics and Dynamics
Heinz Nixdorf Institute
University of Paderborn
Paderborn, Germany, D-33102
Email: brichter@hni.upb.de

Jens Twiefel, Tobias Hemsel, Jörg Wallaschek
Mechatronics and Dynamics
Heinz Nixdorf Institute
University of Paderborn
Paderborn, Germany, D-33102
Email: {twiefel, hemsel, jw}@hni.upb.de

ABSTRACT

This paper presents a general model based on the electromechanical circuit theory. The model is set up as a mechanical equivalent model for base excited systems and describes the behaviour of a piezoelectric element around one resonance frequency which is sufficient for most practical applications.

The model is extended to obtain the influence of geometrical and material properties. The derivated properties are used to describe the parameters of the general model which is easy to handle. Using this model either the calculation of the output power on a specific electric load or the determination of the design of the used piezoelectric element for a needed electric output power is possible. The paper focuses on the design of the ratio of length and width of a piezoelectric bimorph.

The validity of the model is shown by the comparison of computed and experimental results.

NOMENCLATURE

α	Transmission factor between electric and mechanic circuit.
C_p	Capacitance of the piezoelectric element.
c_m	Modal stiffness of the piezoelectric element.
c_p	Inverse internal capacitance.
d_m	Modal damping of the piezoelectric element.
d_{31}	Piezoelectric charge constant of material.
ε_{33}^T	Relative dielectric constant of material.
f_r	Resonance frequency.
h	Height of the piezoelectric beam.
I	Moment of inertia.
k_{31}	Piezoelectric coupling coefficient.
l	Length of the piezoelectric beam.
M	Torsional moment.
m	Mass of the piezoelectric element.
m_0	Mass of the cantilever system.
m_1	Modal mass of the piezoelectric element.
P	Electric output power.
Q_m	Mechanic quality factor of the piezoelectric element.
q	Electric charge.
R_{Load}	Electric load to be powered; considered as resistance.
ρ	Density of material.
s_0	Compliance of cantilever system.
s_1	Compliance of piezoelectric beam.
s_{11}^E	Compliance of piezoelectric material.
t	Time.
v	Velocity.
v_{base}	Excitation velocity of the generator.
w	Width of piezoelectric beam.
Ω	Angular frequency.
ω_r	Angular resonance frequency.
x	Displacement of the generator tip.
Y	Young's modulus.
Y_{el}	Electric admittance $\left\lvert\frac{i}{u}\right\rvert$.
y	Displacement of the generator base.
z_0	First root of the impedance $\frac{F}{v}$ determining resonance.

*Address all correspondence to this author.

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INTRODUCTION

Today, most autonomous low-power electronics are driven by batteries. Unfortunately, battery life does not fit the life-cycle of superior systems. The need for replacement or recharging batteries leads to unwanted effort and costs. Energy harvesting systems could improve autonomous electronics by converting any kind of surrounding energy to needed electrical energy.

In mechanical structures, piezoelectric generators are an appropriate solution to convert ambient vibration energy to electrical energy. Such generators can be used to power online structural health monitoring within manufacturing lines or next generation aircrafts. The optimum design and setup of an energy harvesting system using piezoelectric generators depends on the kind of the surrounding kinetic energy to be exploited (amplitude, frequency) as well as on the electrical application to be powered.

Due to the major dependency of piezoelectric generators on the excitation frequency, it is necessary to use model-based design methods which support the development engineer. To realize such design methods, a good understanding of the piezoelectric system is inevitable. Most modelling techniques use a general generator model. One general model is explained in the beginning of the next section. The advantage of this model is the easy handling. However, the model does not consider material properties or geometric properties directly. Due to the high abstraction level of the used model parameters, the influence of the physical properties is not easy to see.

Particularly, if an optimal geometric dimension of the energy harvesting generator is investigated, this is a drawback. There are some approaches to use material and geometrical data of piezoelectric elements to run optimisations; e.g. Roundy [1] and Sodano [2]. However, these approaches do not use properties that can be found on technical data sheets of the suppliers.

The introduced method in this paper enables the engineer to use published data of the manufacturers. These data can be directly used without further measurements. The presented model uses identification algorithms proposed in Lenk [3]. Furthermore, it is shown, how the parameters of the general model can be identified using admittance measurements on the piezoelectric element. Comparing the identified parameter set with the calculated parameters utilising material and geometrical properties shows the validity of the approach.

Using the model enables the determination of the piezoelectric generator size due to the knowledge of the material properties, the excitement of the generator and the electric application demands. It is shown, that there is an optimal ratio between the length and the width of a piezoelectric bimorph for each power output on a specified resistive load.

MODELING OF A PIEZOELECTRIC BIMORPH

A mathematical model for a piezoelectric generator is described in this section. The general model is suitable to estimate the electric power on a resistance [5]. The model can easily be modified to other electric loads as well. The behaviour of the model is described by several parameters. These parameters can be identified by measurements, but they do neither directly represent the geometric dimensions nor the material properties of the generator. Therefore, an approach to calculate the needed parameters using dimensions and material properties is described in this paper. The approach combines the modelling of a bending beam with the modelling of piezoelectric material properties. This proceeding allows the describtion of the general model parameters using geometric data and material properties according to Lenk [3]. The section closes with an overview of the relation between the properties and the abstract parameters. The benefit of this approach is the possibility for system design optimisation. The knowledge of the influence of geometric and material properties allows the definition of the generators size if the electrical load and power needs are known.

General Model for Piezoelectric Generator Systems

The electro-mechanical model for piezoelectric elements allows an estimation of power output. Due to the electrical and mechanical behaviour of such an element, an analogy is necessary to describe both attributes in one system. The model considers electrical charge as a mechanical displacement and electric voltage as a mechanical force. Because of this analogy, an inductance L is described as a mass m, a capacitance $\frac{1}{C}$ is considered as a spring c and a resistance R is described as a damper d.

Using this analogy, the piezoelectric element is represented in one system which can be expressed as a mechanical or electrical model with a single degree of freedom. Both models characterising the same behaviour are shown in Fig. 1. The model is valid for all base-excited systems with two boundary conditions: The system has to be investigated close to its eigenfrequency and the value of the input velocity $\dot{y}$ has to be small compared to the tip velocity $\dot{x}$ [5]. The model is valid for any piezoelectric generator configuration. Any dielectric losses are neglected in the presented model because they are assumed to be of a very small amount. If needed, they can be easily modelled by an additional resistance in series to the capacitance $C_p = \frac{1}{c_p}$ in the electric circuit.

Examining the mechanical model in Fig. 1(a), it can be seen that the lower part describes the mechanical properties and the upper part represents the electrical attributes of the piezoelectric element. An additional resistive load R_{Load} is included in the upper part modelling the electric application to be powered. The coupling of both parts is done by the lever α indicating the transmission factor. The depicted models can be described by the

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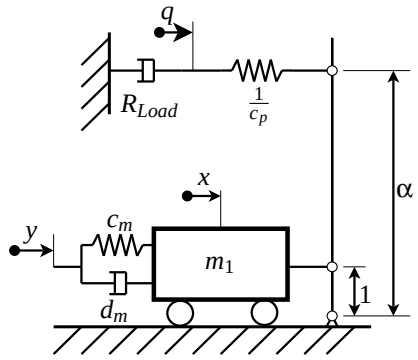
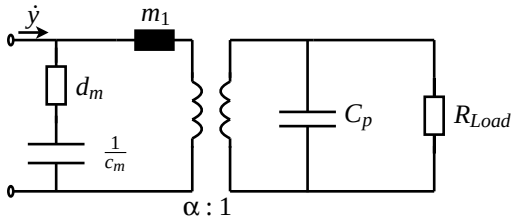


(a) Mechanical Model



(b) Electrical Model

Figure 1. EQIVALENT MODELS FOR BASE EXCITED PIEZOELECTRIC GENERATORS

following mathematical equations:

$$c_m\,(x-y)+d_m\,(\dot{x}-\dot{y})+m_1\ddot{x} = -\alpha R_{Load}\dot{q} \quad (1)$$

$$c_p\,(\alpha\,x-q) = R_{Load}\dot{q} \quad (2)$$

These mathematical equations can be transferred into the Laplace space which yields the equation for $U(s)$. In this term the input velocity $\dot{y}$ at the base of the element is substituted by v_{Base}. This velocity is the input parameter determined by the systems environment. The term $U(s)$ describes the behaviour of the electric voltage in the frequency domain; the supplied power at the resistance R_{Load} can be calculated using Eqn. 4. Replacing s by $j\omega$ yields to a real value $P(\omega)$ due to the conjugate term. Using measurements for identification of the parameter set $\{c_m,\ c_p,\ d_m,\ m_1,\ \alpha\}$, the power at the electric load can be well estimated [5].

$$U(s) = \frac{c_p\,(c_m+d_m\,s)\alpha\,R_{Load}\,v_{base}}{s\,c_p\,R_{Load}\,\alpha^2+(c_m+s\,(d_m+m_1\,s))\,(c_p+s\,R_{Load})} \quad (3)$$

$$P(s) = U(s)\left(\frac{U(s)}{R_{Load}}\right)^* \quad (4)$$

General Cantilever Beam Model

To obtain a model based on physical properties, the system has to be defined in more detail than in the general model. The piezoelectric generator uses mechanical system vibrations to gain electric power. Due to low frequencies and low forces of most vibrations, it is reasonable to use a piezoelectric system configuration which is a simple base-excited cantilever beam.

To calculate the dynamic behaviour of the cantilever beam, the equation of motion for an elastic beam is used. Because the introduced model for a piezoelectric element uses velocities, the behaviour is described according to $v(x,t)$ instead of the deflection $w(x,t)$. The beam mass is defined by $m'l := \rho whl$ and the compliance of the beam is described as $s'l^3 := \frac{1}{YI}l^3$ (cmp. Eqns. 17-18). Shear forces F and motions v as well as torsional moments M and angular velocities Ω are effective in the cantilever model at both ends of the beam. The equation of motion Eqn. 5 can be achieved according to Euler-Bernoulli using $A(x,t) := a(x)cos(\omega t)$ to seperate time and location dependend expressions.

$$\frac{d^4\,v}{dx^4} - \beta^4 v = 0 \text{ with } \beta^4 \equiv \omega^2\,m'\,s' \quad (5)$$

The differential equation Eqn. 5 can be solved using the following Rayleigh-functions as suggested by Lenk [3]:

$$S(x) = \frac{1}{2}(\sinh x+\sin x) \quad (6)$$

$$C(x) = \frac{1}{2}(\cosh x+\cos x) \quad (7)$$

$$s(x) = \frac{1}{2}(\sinh x-\sin x) \quad (8)$$

$$c(x) = \frac{1}{2}(\cosh x-\cos x) \quad (9)$$

Using the Rayleigh-functions and the derivations of v, Ω and M leads to the description of the elastic beam as following. The parameters α_n are calculated using the boundary conditions at both ends of the beam.

$$v(x) = \alpha_1 C(\beta x)+\alpha_2 S(\beta x)+\alpha_3 c(\beta x)+\alpha_4 s(\beta x) \quad (12)$$

$$\Omega(x) = \beta(\alpha_1 s(\beta x)+\alpha_2 C(\beta x)+\alpha_3 S(\beta x)+\alpha_4 c(\beta x)) \quad (13)$$

$$M(x) = -\frac{\beta^2}{j\omega s'}(\alpha_1 c(\beta x)+\alpha_2 s(\beta x)+\alpha_3 C(\beta x)+\alpha_4 S(\beta x)) \quad (14)$$

$$F(x) = -\frac{\beta^3}{j\omega s'}(\alpha_1 S(\beta x)+\alpha_2 c(\beta x)+\alpha_3 s(\beta x)+\alpha_4 C(\beta x)) \quad (15)$$

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$$\begin{pmatrix} v_0 \\ \Omega_0 \\ M_0 \\ F_0 \end{pmatrix} = \begin{pmatrix} C & -\frac{1}{\eta}S & \frac{1}{jz_0 l}c & \frac{1}{jz_0\eta}s \\ -\frac{\eta}{l}s & C & -\frac{\eta}{jz_0 l^2}S & -\frac{1}{jz_0 l}c \\ jz_0 lc & -\frac{jz_0 l^2}{\eta}s & C & \frac{l}{\eta}S \\ jz_0\eta S & -jz_0 lc & \frac{\eta}{l}s & C \end{pmatrix} \begin{pmatrix} v_1 \\ \Omega_1 \\ M_1 \\ F_1 \end{pmatrix} \tag{10}$$

$$\begin{pmatrix} F_0 \\ M_0 \\ F_1 \\ M_1 \end{pmatrix} = \frac{jz_0}{c^2 - sS} \begin{pmatrix} -\eta(SC - sc) & -l(cC - s^2) & \eta S & -lc \\ -l(cC - s^2) & -\frac{l^2}{\eta}(cS - sC) & lc & -\frac{l^2}{\eta}s \\ -\eta S & -lc & \eta(SC - sc) & -l(cC - s^2) \\ lc & \frac{l^2}{\eta}s & -l(cC - s^2) & -\frac{l^2}{\eta}(sC - cS) \end{pmatrix} \begin{pmatrix} v_0 \\ \Omega_0 \\ v_1 \\ \Omega_1 \end{pmatrix} \tag{11}$$

Considering the length l of the cantilever, the argument of the Rayleigh-functions is βl. Regarding this term, the following abbreviations are used. The compliance of the whole beam is described as s_0 and the mass of the system as m_0. These two terms yield the eigenfrequency ω_0 of the cantilever. The impedance z_0 can also be concluded.

$$\beta l = l\sqrt{\omega\sqrt{m's'}} = \sqrt{\omega\sqrt{m'ls'l^3}} = \sqrt{\frac{\omega}{\omega_0}} = \eta \tag{16}$$

$$m_0 = m'l = \rho whl \tag{17}$$

$$s_0 = s'l^3 = \frac{l^3}{YI} \tag{18}$$

$$\omega_0 = \frac{1}{\sqrt{m_0 s_0}} \tag{19}$$

$$z_0 = \sqrt{m_0 s_0} = \frac{1}{\omega_0\ s_0} \tag{20}$$

Using these abbreviations leads to the matrix in Eqn. 10. Rearranging the matrix elements yields the impedance matrix Eqn. 11. Both matrices describe the waveguide of the cantilever of the length l using η as argument of the Rayleigh-functions Eqns. 6-9. The mathematical model of the waveguide using correct boundary conditions describes the dynamic behaviour of a simple cantilever beam.

Cantilever Model for Piezoelectric Bimorphs

The general cantilever model does not consider the piezoelectric effect which is the generation of charges if the element is loaded. One way to describe the piezoelectric effect for bending structures is described in Eqns. 21, 22 using the abbreviation k_{31}^2; cp. [4]. The model describes the relationship between the mechanic parameters tension T, strain S and the electric parameters dielectric displacement D, electrical field E. The modelled bending structure is assumed to be a piezoelectric bimorph having two layers of the of the same height. A shim or adhesive layers are neglected.

$$T_1 = \frac{1}{s_{11}^E(1-k_{31}^2)}S_1 - \frac{1}{d_{31}}\left(\frac{k_{31}^2}{1-k_{31}^2}\right)D_3 \tag{21}$$

$$E_3 = -\frac{1}{d_{31}}\left(\frac{k_{31}^2}{1-k_{31}^2}\right)S_1 + \frac{1}{\varepsilon_{33}^T(1-k_{31}^2)}D_3 \tag{22}$$

$$k_{31}^2 = \frac{d_{31}^2}{e_{33}^T s_{11}^E} \tag{23}$$

The piezoelectric equations can be rewritten to show the dependency of torsional moment, angular velocity, electric voltage and electric current using Eqns. 24-27. The equations are valid for two-layer elements with identical layer width.

$$M = -w\int_{-\frac{h}{2}}^{\frac{h}{2}} T_1 x dx \tag{24}$$

$$S_1 = h\frac{\Omega}{j\omega l} \tag{25}$$

$$u = \int_0^{\frac{h}{2}} E_3 dx \tag{26}$$

$$i = 2j\omega D_3 \Delta A \tag{27}$$

Using the equations for M, Ω, u and i yields the matrix Eqn. 28. This matrix describes the behaviour of a piezoelectric beam as depicted in Fig. 2. In the following, ΔA describes the cross-sectional area of the beam and $I = \frac{wh^3}{12}$ defines the moment of inertia of the beam.

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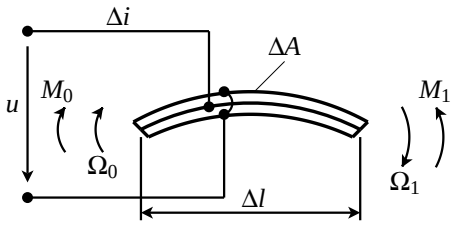

Figure 2. ELEMENT OF PIEZOELECTRIC BEAM.

$$\begin{pmatrix} \Delta i \\ M \end{pmatrix} = \begin{pmatrix} j\omega\Delta C_b & \frac{1}{\alpha'} \\ \frac{1}{\alpha'} & -\frac{1}{j\omega\Delta s_0} \end{pmatrix} \begin{pmatrix} u \\ \Delta\Omega \end{pmatrix} \tag{28}$$

$$\Delta C_p = 4\varepsilon_{33}^T(1-k_{31}^2)\frac{\Delta A}{h} \tag{29}$$

$$\Delta s_0 = \frac{\Delta s'}{YI} = \frac{s_{11}^E}{I}\left(\frac{1-k_{31}^2}{1-\frac{3}{4}k_{31}^2}\right)\Delta l \tag{30}$$

$$\alpha' = \sqrt{\frac{s_0}{C_b}\frac{1-k_B^2}{k_B^2}} \tag{31}$$

Now we consider the infinitesimal beam elements of the generalised cantilever model. These elements are expanded to indicate the piezoelectric effect. A schematic view to the expanded system is depicted in Fig. 3. The system consists of two transformation blocks: The first block describes the waveguide of the beam determined by the general cantilever model with η and z_0. Because no boundary conditions have been defined yet, all eight coordinates of the mechanical system has to be represented. The mathematical model is shown in Eqn. 10. The second block specifies the coupling of the mechanical and the electrical part as described by the piezoelectric model (mathematically modeled in Eqn. 28). Replacing the torsional moments $M_{0,1}$ by $M'_{0,1}$ in Eqn. 10 enables the link between both mathematical models.

For the mathematical model, the boundary conditions of the beam have to be defined. A piezoelectric generator works like an accelerometer. The generator is accelerated by the mechanical vibrations of the host system. Due to the base excitation by the acceleration a and the fixed support on one side of the cantilever, the boundary conditions are shown in Eqns. 33-34. According to Eqn. 10 and the boundary conditions, the piezoelectric generator can be mathematically described by Eqn. 37 after elimination of the motion v_1 using Eqn. 36. The equation of the moment M_0 is not of interest to determine the energy output of the generator.

The complete mathematical model for a dynamic piezoelectric beam is obtained by the coupling of Eqn. 28 and Eqn. 37 substituting the vector $\begin{pmatrix} \Omega_1 & M'_1 \end{pmatrix}^T$ in both equations. It can be seen that Fig. 3 represents the resulting mathematical model.

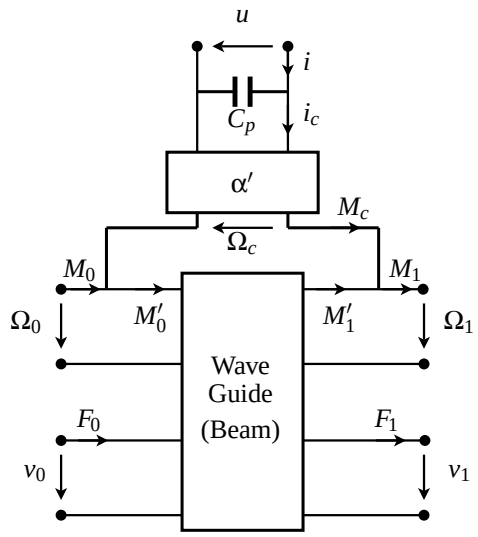

Figure 3. SCHEMATIC VIEW OF THE PIEZOELECTRIC BEAM; CP. [3].

$$v_0 = \frac{a}{j\omega}, \quad F_0 = ma \tag{32}$$

$$\Omega_0 \equiv 0, \quad M_1 \equiv 0, \quad q_1 \equiv 0 \tag{33}$$

$$M'_1 = -M_c, \quad \Omega'_1 = \Omega_c \tag{34}$$

$$\Omega_0 \equiv 0 = -\frac{\eta}{l}sv_1 + C\Omega_1 - \frac{\eta}{jz_0l^2}SM'_1 - 0 \tag{35}$$

$$\Rightarrow v_1 = \frac{Cl}{\eta s}\Omega_1 - \frac{1}{jz_0l}\frac{S}{s}M'_1 \tag{36}$$

$$\begin{pmatrix} v_0 \\ F_0 \end{pmatrix} = \begin{pmatrix} \frac{l}{\eta}\left(\frac{C^2-sS}{s}\right) & \frac{1}{jz_0l}\left(\frac{cs-CS}{s}\right) \\ jz_0l\left(\frac{SC-sc}{s}\right) & \frac{\eta}{l}\left(\frac{s^2-S^2}{s}\right) \end{pmatrix} \begin{pmatrix} \Omega_1 \\ M'_1 \end{pmatrix} \tag{37}$$

Derivation of Parameters for the General Model

The cantilever model for piezoelectric bimorphs allows the description of the parameters in the general model by geometric and material properties. Due to the same behaviour of the general piezoelectric model and the piezoelectric beam model, the parameters of the general model can be achieved by the physical properties of the beam model. Lenk [3] explains how these parameters are obtained. The electrical parameters can be derived from the bimorph model (see Eqn. 28 to Eqn. 31) whereas the mechanical properties are extracted basically from the cantilever model. To convert as much mechanical energy into electric energy as possible, the piezoelectric generator should be operated in resonance. For each resonance frequency, the ratio of the impedance $\frac{F_0}{v_0} = z$ (derived by Eqn. 38) equals zero. In our model, the lowest resonance frequency occurring at the first root of z is considered.

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$$z = jz_0\eta\frac{1+\cosh\eta\cos\eta}{\sinh\eta\cos\eta-\cosh\eta\sin\eta} = 0 \quad (38)$$

$$\Rightarrow \cosh\eta\cos\eta = -1 \quad (39)$$

$$\Rightarrow \eta_1 = 1.875 \quad (40)$$

The parameters s_1 and m_1 yield from the derivation of the function $z(\omega)$ at the lowest resonance eigenfrequency ω_r. The parameter s_1 can also be derived from Eqns. 16-18. The damping results from Eqn. 43, where Q_m indicates the quality factor of the used piezoelectric ceramics.

$$m_1 = \frac{m}{4} \quad (41)$$

$$s_1 = \frac{1}{\omega_1^2 m_1} = \frac{4}{\eta_1^4}s_0 = 0.971\frac{s_0}{3} \quad (42)$$

$$d_1 = \frac{1}{Q_m\omega_r s_1} \quad (43)$$

$$\omega_1 = \sqrt{\frac{1}{ms_1}} \quad (44)$$

The abstract model parameters of the simple generalised piezoelectric element introduced at the beginning of the paper can now be described by the presented equations. To get a better overview, the transformations of the geometrical and material beam properties to the needed parameters are shown in Tab. 1. The calculated parameter set is valid around the first resonance frequency. For higher frequencies, the value of η for the next root of z (Eqn. 38) has to be calculated.

Furthermore, the couple factor α is of importance. This factor is already shownin Eqn. 31. The frequency dependency of the transformer element is incorporated for any frequencies in the equation 45. Considering that the first resonance is investigated, Eqn. 46 follows using Eqn. 40.

$$\alpha = \alpha'\frac{l}{\eta}\left(\frac{\cosh\eta\sin\eta-\sinh\eta\cos\eta}{\sinh\eta\sin\eta}\right) \quad (45)$$

$$\Rightarrow \alpha = \alpha'\frac{l}{\eta_1}\tan\frac{\eta_1}{2} = 0.727l\alpha' \quad (46)$$

The parameters in Tab. 1 can directly be used in the general piezoelectric model. Of course, the use of parameters in Tab. 1 is only valid for piezoelectric bimorphs with the same boundary conditions as explained before operating at the first eigenmode.

The yield mathematical derivations are checked on plausibility. The mass m_1 describes the effective mass which has to be considered. The stiffness c_m is also easy to follow. Investigating Eqn. 42 shows a term in the following form; the compliance has changed to a stiffness by using the reciprocal value: $c_1 = \frac{I}{s_{11}^E}\frac{1}{3.884\ k_1}$. The first fraction shows obviously the flexural rigidity YI and the second term is related to the piezoelectrical properties. The damping d_m is concluded from the quality factor Q_m. The capacitance C_p is the usual equation for a capacitor, also with an additional term due to the coupling to a mechanical system.

Table 1. CALCULATION OF THE PARAMETER SET

Model Parameter	Equation Number	Mathematical Derivation
m_1	41	$\frac{\rho lwh}{4}$
$c_m = \frac{1}{s_1}$	42	$\frac{wh^3}{3.884 s_{11}^E l^3 k_1}$
d_m	43	$\frac{wh^2}{4Q_m l}\sqrt{\frac{\rho}{0.971 s_{11}^E k_1}}$
α	46	$\frac{wh}{0.727l\sqrt{\frac{3k_2 s_{11}^E}{\varepsilon_{33}^T}}}$
C_p	cmp. 29	$4\varepsilon_{33}^T\frac{lw}{h}k_1$
k_B^2		$\frac{3}{4}k_{31}^2$
k_1		$\left(\frac{1-k_{31}^2}{1-k_B^2}\right)$
k_2		$\left(\frac{1-k_B^2}{k_B^2}\right)$

PARAMETER IDENTIFICATION USING MEASUREMENTS

The identification of the model parameters is usually done with the help of measurements. A fast method for the identification is presented in the first part of this section utilising admittance measurements. In the second part it is shown that the general generator model using the identified parameters is able to describe the behaviour of a piezoelectric bending device.

The determined parameter set is later used for comparison with the calculated data set depending on material and geometrical properties to validate the paper's approach.

Theory of the used Identification Method

The parameters of the general piezoelectric model for base-excited systems are usually identified by measurements as described at the beginning of this paper. Due to easier measurement methods an actuator model instead of a generator model is used to determine the parameter set of the piezoelectric element. The only difference between the actuator model and the

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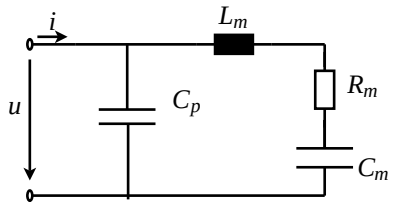
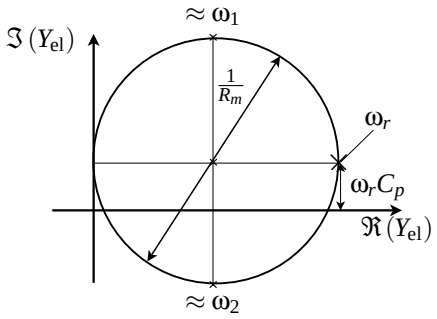


(a) Reduced model for evaluation of admittance measurements



(b) Ideal frequency response locus

Figure 4. PARAMETER IDENTIFICATION

generator model are the boundary conditions. The generator is base-excited by mechanical oscillations whereas the actuator is driven by an electric voltage. It is shown, that the identified data set is valid.

The identification itself uses admittance measurements. First of all, the model can be reduced in a simple electric part using the electro-mechanical analogy [6]; the resulting electric circuit is depicted in Fig. 4(a). Using the reduced model, a frequency response locus of the piezoelectric element can be constructed. Eqn. 47 describes the locus on the complex plane with radius r. This approach is valid for small values for the internal capacitance C_p. The theoretical expected locus is shown in Fig. 4(b).

$$Y_{el} = \frac{i}{u} = \alpha^2 \frac{d_m - jX_m}{d_m^2 + X_m^2} + j\omega C_p \text{ with } i := \dot{q} \tag{47}$$

$$X_m = \omega m_1 - \frac{c_m}{\omega} \tag{48}$$

$$r = \frac{\alpha^2}{2d_m} = \frac{1}{2R_m} \tag{49}$$

The electrical admittance measurement of the piezoelectric bimorph will yield a locus similar to the depicted one. The locus allows the identification of five parameters which are components of the condensed model of Fig. 4(a). The equivalent of the mechanical damping (R_m) is described by the diameter of the locus. The mechanical damping is easily identified as shown in Eqn. 50 evaluating the admittance at the series resonance frequency ω_r, because dielectric losses can be neglected. At the same point, the value for C_p can be determined as shown in Eqn. 51. The mass described by L_m is achieved using ω_1 and ω_2. The stiffness of the spring is identified by the general term for resonance as shown in Eqn. 53 [6].

$$\frac{d_m}{\alpha^2} = R_m = \frac{1}{max(\Re(Y_{el}))} \tag{50}$$

$$C_p = \frac{max(\Im(Y_{el})) - min(|\Im(Y_{el})|)}{2\omega_r} \tag{51}$$

$$\frac{m_1}{\alpha^2} = L_m = \frac{R_m}{\omega(max(\Im(Y_{el}))) - \omega(min(|\Im(Y_{el})|))} \tag{52}$$

$$\frac{\alpha^2}{c_m} = C_m = L_m \omega_r^2 \tag{53}$$

Finally, the parameter α has to be determined which is not possible using only the electrical admittance locus. To achieve the parameter, an additional measurement of the mechanical admittance has to be done. The parameter α can be calculated by the ratio of the electrical to the mechanical admittance as shown in Eqn. 54 [5]:

$$\alpha = \frac{max(\Re(Y_{el}))}{max(\Re(Y_{mech}))} = \frac{1}{R_m \; max(\Re(Y_{mech}))} \tag{54}$$

Experimental Identification

To validate the calculated parameters, the behaviour of the piezoelectric elements is measured and compared to the behaviour of the model. For our investigations we selected a piezoelectric bimorph using the material VIBRIT 1876. The properties of the selected element can be seen in Tab. 2. We measured the mechanical and electrical admittances of three different brand-new elements of this type. The parameters are identified using the described Eqns. 50-53. The measurements show a variance of the beams depending on the material tolerances as expected.

The validation of the identified parameters is shown by comparison of the measured frequency responces and the modeled frequency response using one identified parameter set. The identified parameters shown in Tab. 3 uses the data set of the median beam. The frequency responses are shown in Fig. 5 with the median curve coloured in blue (dark line). The model behaviour is indicated by lines; all measurements are depicted as dots indicating measured points. As can be seen, the measurements do fit the model very good, especially around the resonance frequency. A minor deviation can be found at the anti-resonance.

Considering the piezoelectric element as a generator, the power over an electric load is of interest. Using Eqn. 4, the

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Table 2. GEOMETRIC AND MATERIAL PROPERTIES OF PIEZOELECTRIC BIMORPH CONSISTING OF VIBRIT 1876.

Property	(Characteristic) Value
l $[mm]$	39.5 (free length)
w $[mm]$	7.2
h $[mm]$	0.8
ρ $[kg/m^3]$	8000
k_{31}	0.38
s_{11}^E $[m^2/N]$	15.8 10^{-12}
ε_{33}^T $[F/m]$	61950 10^{-12}
Q_m	45

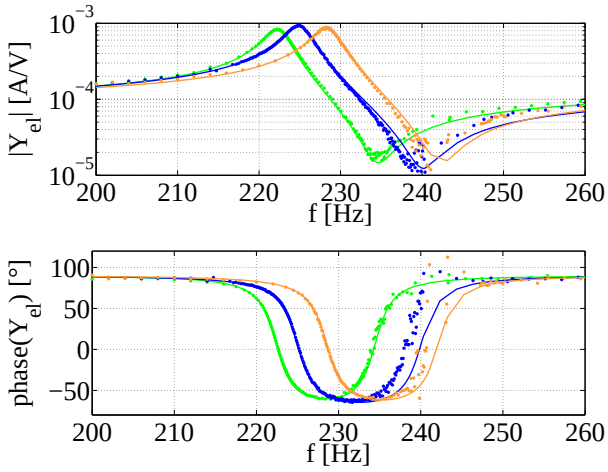
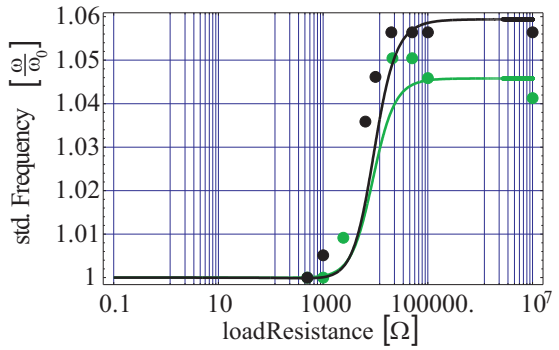
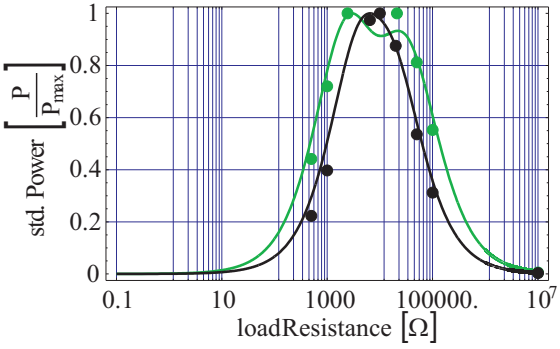


Figure 5. FREQUENCY RESPONSE OF PIEZOELECTRIC ELEMENTS. COMPARING MEASUREMENTS AND MODEL.

expected power output can be calculated with the identified parameters. A comparison of power measurements and modelled data shows a good matching as depicted in Fig. 6 [5]. Again, measured data is indicated by dots in both diagrams. Fig. 6(a) shows the change of frequency at maximum output power for changing electrical load resistance. In Fig. 6(b), the normalised power output over a load resistance is shown at resonance frequency. It depicts the maximal power output for each connected electric load for constant excitation v_{base}. Considering the shown plots, it can be concluded that the explained identification method is valid.



(a) Freq vs. load at maximal power



(b) Output power vs. load at resonance

Figure 6. COMPARISON MEASUREMENT AND MODEL (black - excitation level 10V, green - excitation level 0.5V); cp. to [5].

MODEL VALIDATION

In the previous section it has been shown that the generalised model using parameters on an abstract level can be used to determine the behaviour of the piezoelectric generator. It is also made clear how the parameters of the general model can be obtained by admittance measurements.

The validity of the model using geometric and material properties is checked by measurements of the frequency response $\left|\frac{i}{u}\right|$. The selected piezoelectric bimorph has the material and geometrical properties as shown in Tab. 2.

The associated plots to the admittance measurements are shown in Fig. 7. The green, blue and orange lines show the model response with identified parameters whereas the red lines show the model response using calculated parameters; cp. Fig. 5. Considering the high material tolerances for piezoelectric elements, an acceptable matching between measurements and the model is obtained. A frequency tolerance of less than 10% can be determined in Fig. 7(a) due to the simplifications done by the modelling, e.g. the real system has always an adhesive layer between both piezoelectric layers. Additionally, the active piezoelectric layers of the bimorph does not cover the whole free length of the cantilever beam as assumed in the model. Furthermore, the freqeuncy shift may be caused by fixed support of the bending

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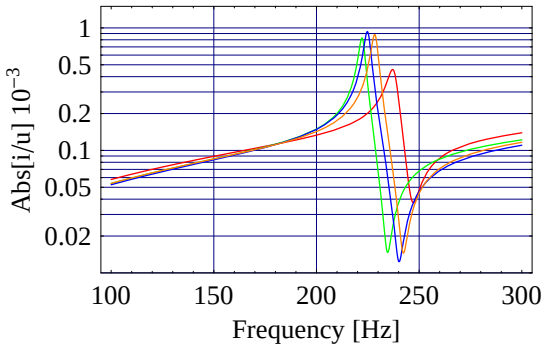
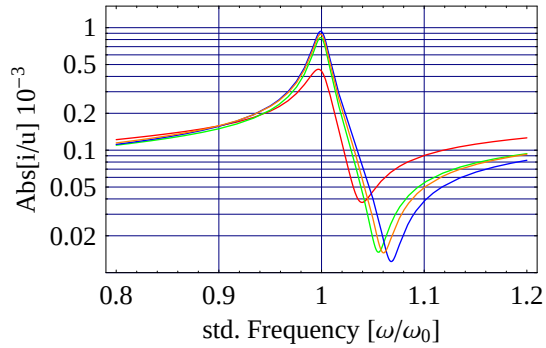


(a) Plot of the frequency responce.



(b) Plot of the frequency responce normed to resonance frequency.

Figure 7. FREQUENCY RESPONSE: COMPARING MEASUREMENT AND MODEL UTILISING GEOMETRICAL AND MATERIAL PROPERTIES.

Table 3. IDENTIFIED AND CALCULATED PARAMETER SETS FOR SELECTED PIEZOELECTRIC BIMORPH.

Parameter	Identified value	Calculated value
$m\ [kg]$	$4.097\ 10^{-4}$	$4.550\ 10^{-4}$
$c_m\ [N/m]$	818.54	1015.83
$d_m\ [N\ s/m]$	$8.75\ 10^{-3}$	$15.11\ 10^{-3}$
$\alpha\ [N/V]$	$2.84\ 10^{-3}$	$2.53\ 10^{-3}$
$C_p\ [F]$	$71.16\ 10^{-9}$	$84.53\ 10^{-9}$
$f_r\ [Hz]$	224.96	237.80

beam which is not as ideal as modelled.

The calculated parameters using material and geometric properties are compared with the identified parameters in Tab. 3. It can also be seen in the table, that the resonance frequency f_r is estimated too high, because the stiffnesses yields a too high value.

DESIGN OF A PIEZOELECTRIC GENERATOR

The validation shows a usable model, although stiffnesses are calculated to high using the material properties and geometrical dimensions of a piezoelectric bimorph to determine the parameters using the equations of Tab. 1. Because the model determines power output very well, it must be possible to achieve the best dimensions of the beam for a given electric load and power demand.

The design of a piezoelectric generator depends on the application needs. The application specifies the electrical application with power demand P_d and load resistance R_{Load} as well as the superior system which defines input velocity v_{base} and input frequency ω. Assuming the application is known, the only free parameters in Eqn. 4 are the material properties and the geometry represented by l, w and h. The material properties should be preliminary selected by the engineer. If the power output is too small, another material data set may be selected, so the resulting geometric dimensions are the only free parameters in the equation. A two-dimensional optimisation problem is achieved, if the height is hold constant. Variing the length and width of a piezoelectric bender yields to a plot as shown in Fig. 8(a)

To get proper solutions, Eqn. 4 is derived to ω. Calculating the roots of this term yields all combinations of w and l, at which the power output is maximal (with $\frac{d^2P(\omega)}{d\omega^2} < 0$). The smallest possible combination of width and length is achieved by comparing the resulting roots with the power output defined in Eqn. 4. Of course, not only optimal ratios of w and l will yield enough power output; cp. Fig. 8(b).

The design method is shown in detail using the following example application. The generator is excited by the environment with an input velocity $v_{base} = 15\ \frac{mm}{s}$ at $\omega = 2\ \pi\ 235\ Hz$. The electric application consumes 4 mW at a resistive load of 80 $k\Omega$. VIBRIT 1876 is considered as piezoelectric material. The height of the beam is set to 0.8 mm. The resulting power plot Fig. 8(b) shows enough power output to satisfy the application demands. The derivation of the power term leads to the dashed red curve showing all optimal combinations of the width and the length of the piezoelectric beam. The green coloured area defines all valid combinations to gain the desired electric power. The smallest optimal combination satisfying power demands is shown by the intersection point of the red and the green line at $l = 40.4\ mm$ and $w = 2.6\ mm$. Using the plot Fig. 8(b) enables the selection of a proper piezoelectric cantilever beam.

CONCLUSION

The previous sections present a sufficient model using material and geometrical properties to describe the behaviour of a piezoelectric generator. The possibility to use the easy to handle generalized piezoelectric generator model is a great advantage of the shown method, because additional masses for resonance

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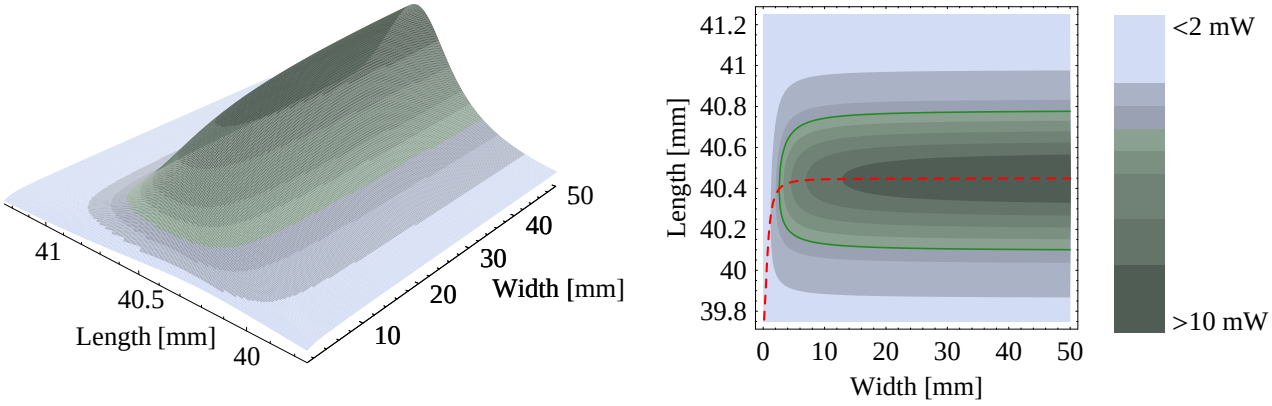

(a) Power Output depending on width and length.

(b) Power Output. The dashed line shows all combinations of the length and width where power output is optimal. The area encircled by the solid line specifies the region where power output satisfies power demand of 4 mW.

Figure 8. POWER OUTPUT DEPENDING ON GENERATOR DIMENSIONS FOR SAMPLE APPLICATION.

tuning or other electric loads can be easily adapted, for instance. The abstract parameters are replaced by related geometrical and material properties. It is shown that the model matches measurements in an acceptable way if the model works in the resonance frequency, considering the tolerances of material properties.

The model can be used to determine the theoretic optimal design for a piezoelectric bending beam if either the piezoelectric element or the electric load R_{Load} and power demand P_d are known. Especially the last case is of interest for a best possible design of piezoelectric generators. A method to determine an appropriate and optimal ratio of the length and width of the piezoelectric beam is shown. The smallest possible optimal combination of these two geometrical properties satisfying application demands yields the minimum beam size for a given material properties set and a selected beam height.

In a further step the electric load using an alternate current to a direct current load. This load type will better match a real application which may be powered by energy harvesting systems.

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